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Analytical limit load predictions in heterogeneous welded single edge notched tension specimens

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Abstract

The integrity assessment of defected welds is dependent on accurate estimations of their load carrying capacity. As welds consist of variable microstructures, a large degree of heterogeneity is to be expected. The variation of constitutive properties within the weld influences the deformation patterns around the crack and, as a consequence, the load bearing capacity of the joint. Constitutive heterogeneity is simplified in standardized assessments in order to facilitate the analysis and reduce the complexity of its required input. However, these weld simplifications may lead to inaccurate assessments with unknown errors. This motivates the work of the authors, which aims to include the effects of weld heterogeneity into integrity assessment procedures. The presented paper focuses on the prediction of limit load, which allows to calculate the structure's proximity to plastic collapse. Simplified theorems have been developed to identify lower and upper bound values of limit load. This work explores the predictive accuracy of various methods to estimate the limit load of heterogeneous welds, including lower and upper bound theorems. A parametric study involves 2D plane strain simulations of single-edge notched tension (SE(T)) specimens. Welds consisting of two regions of different material properties (at the root and at the cap) are introduced. The obtained estimations of limit load are then compared against the simulated limit loads.

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Keywords: Weld heterogeneity; SE(T); Limit loads; upper and lower bound; slip lines

1. Introduction

Estimation of load carrying capacity of a structure plays a pivotal role in its integrity assessment. In order to predict the load carrying capacity, limit analysis is performed. Focusing on global collapse in this paper, limit analysis is the

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estimation of a critical load at which the plastic region has extended over the entire cross section and there is an unconstrained plastic flow. Beyond this point, the load does not increase assuming perfectly plastic material. This critical load is termed as 'limit load'. Analytical limit theorems for plasticity provide relatively easier estimations of upper and lower bound limit loads without any complexities of experiments and simulations. These theorems are based on severe assumptions and it becomes difficult to apply in case of heterogeneous material. A welded structure is a perfect example of a heterogeneous region which consists of different material properties in base material, heat affected zone and weld region. Several researchers have assessed the analytical limit load solutions of a notched weld subjected to tension and bending loads. Kumar, German et al. (1981), Milne, Ainsworth et al. (1988) and Miller (1988) presented lower bound limit load solutions of several notched specimen configurations. Joch, Ainsworth et al. (1993) and Hao, Cornec et al. (1997) considered deformation fields of a notched welded panels to derive upper bound limit load solutions. The weld mismatch effects were included in the developed upper bound limit load equations by several researchers like Alexandrov, Chicanova et al. (1999), Kozak, Gubeljak et al. (2009) and Kim and Schwalbe (2001). However, the derivations of analytical limit load estimations which incorporates the material property variations within the weld, is missing. It is evident from previous works of Hertelé, O'Dowd et al. (2015), Zerbst, Ainsworth et al. (2014) and Naib, De Waele et al. (2018) that the heterogeneity within a weld affects the crack behavior which in turn affects the limit load of the structure. In this research, a lower bound limit load equation is derived for a heterogeneous weld in a Single Edge notched Tensile (SE(T)) specimen. SE(T) specimens are suitable to assess defects when they are subjected to high deformations under low constraint conditions. Along with lower bound theorem, upper bound equations developed for mismatched welds are utilized to assess the limit loads of SE(T) specimens. The analytical results are compared with limit loads obtained from Finite Element (FE) simulations. In this paper, *section 2* describes the analytical lower and upper bound equations which are used to obtain limit load of the SE(T) specimens having heterogeneous welds. *Section 3* details the numerical model used to validate analytical equations. *Section 4* enunciates results and discussions and *section 5* concludes the research paper.

Nomenclature

σ	Equivalent stresses in a homogeneous body due to applied forces *(N/mm²)*
σ_{y1}	Yield stress of a homogeneous body *(N/mm²)*
σ_{ym}	Yield stress of the mismatched weld region *(N/mm²)*
σ_{yb}	Yield stress of the base material *(N/mm²)*
A	Area of the body *(mm²)*
a	Notch depth *(mm)*
B	Thickness of the SE(T) specimen *(mm)*
b	Ligament width *(W-a) (mm)*
H_r	Half thickness of the weld *(mm)*
L	Daylight length of the SE(T) specimen *(mm)*
M_{eq}	Mismatch ratio ($M_{eq} = \sigma_{yw} / \sigma_{yb}$)
M_r	Mismatch ratio of the root
M_c	Mismatch ratio of the cap
P	Applied load *(kN)*
P_{LB}	Lower bound limit load *(kN)*
P_{UB}	Upper bound limit load *(kN)*
r	Radius of the notch tip *(mm)*
W	Width of the SE(T) specimen *(mm)*
W_r	Width of the weld root *(mm)*
W_c	Width of the weld cap *(mm)*

2. Analytical limit load estimations

2.1. Lower bound limit load

Lower bound theorem states that 'in an elastic-fully plastic body, when the stresses are in equilibrium with the boundary conditions and the equivalent stress does not exceed yield stresses, then the maximum load estimated will be lower than the actual load required to cause plastic collapse' (Hill (1951), Drucker, Prager et al. (1952)). Accordingly, for the homogeneous SE(T) specimen shown in *figure 1,* the specimen collapses when the equivalent stress σ reaches yield σ_y. If, $P = P_{LB}$ and when $\sigma = \sigma_y$,then the limit load of the homogeneous SE(T) sample can be defined as follows:

$$P_{LB} = c.\sigma_y.A = c.\sigma_y.\mathrm{B}.\mathrm{b} \tag{1}$$

where, c is a factor that depends on the yield criterion (1.155 assuming the von Mises criterion). B is considered to be unity (assuming plane strain conditions). Similarly, for a welded SE(T) specimen *(figure 2)* having different material properties in the root and the cap region, the equation for lower bound limit load can be modified as:

$$P_{LB} = c.B.(b_1.\sigma_{y1} + b_2.\sigma_{y2}) \tag{2}$$

where, b_1 and b_2 are the widths of the cap and root as shown in *figure 2* and σ_{y1} and σ_{y2} are the yield strengths of the cap and the root of the weld respectively. It is important to realize that heterogeneous welds may show different locations of failure, depending on crack dimensions and weld strength mismatch ratios. Whereas Eq. (1) expresses collapse entirely confined within the weld, a strong welded connection may fail in the base metal. A third possible failure trajectory for the weld shown in *Figure 2* is collapse in a slip line originating from the crack tip, then escaping the weld root region and heading into the base metal. The actual lower bound limit load is then the minimum of lower bounds associated with each of these three failure modes (Eq. (2) for confined yielding, and similar equations for the other failure modes). The limit load of a welded connection is often expressed relative with respect to the base metal limit load. This ratio is the "equivalent strength mismatch" (with respect to base metal yield strength) of a hypothetical homogeneous connection that would have the same limit load. Expressing the minimum of three above mentioned limit loads (including equation (1) in terms of equivalent mismatch (M_{eq}) as indicated by Kim and Schwalbe (2001) and Hertelé, De Waele et al. (2014) eventually leads to the expression:

$$M_{eq} = \frac{P_{LBm}}{P_{LBb}} = \min\left(\frac{W}{W-a}; \frac{b_1}{b_1+b_2} M_r + \frac{b_2}{b_1+b_2} \min(1, M_c)\right) \tag{3}$$

Note that the factor c relating to the yield criterion has vanished. The material property variations (σ_{y1} and σ_{y2}) in root and cap along with the SE(T) thickness, crack depth, and location of the root-to-cap interface, will have an effect on the lower bound limit load estimate.

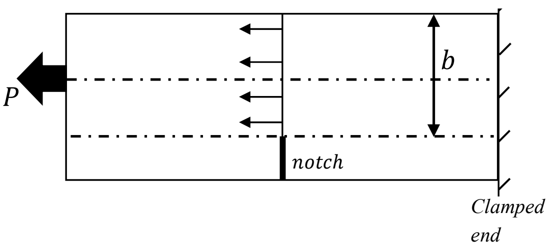

Figure 1: Statically admissible stress field in homogeneous SE(T) specimen

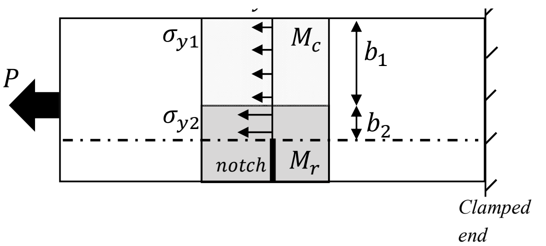

Figure 2: Statically admissible stress field in heterogeneous SE(T) specimen

2.2. Upper bound limit load

The upper bound limit load theorem states that in an elastic-perfectly plastic body having a kinematically admissible velocity field, then the maximum load estimated will be higher than the actual load required to cause plastic collapse.

In this study, it is assumed that straight slip lines originate from the crack tip at an angle of 45° with respect to the loading direction. This is the theoretical slip line solution for a homogeneous SE(T) configuration, but need not necessarily be the correct slip line for a heterogeneous connection. The upper bound limit load is determined in terms of equivalent mismatch (M_{eq}) i.e. the ratio of mismatched limit load of SE(T) specimen to the limit load of the SE(T) specimen with homogeneous base material, which is as expressed in lower bound solutions in *section 2.1*. Equivalent mismatch is calculated by the equation (13) as shown in the paper of Hertelé, De Waele et al. (2014). It is the average of the weld strength mismatch level measured along the portion of the slip line (OF) (shown in *figure 3*) located in the weld.

$$M_{eq} = \frac{\int_{OF} M(s)ds}{\| OF \|}$$

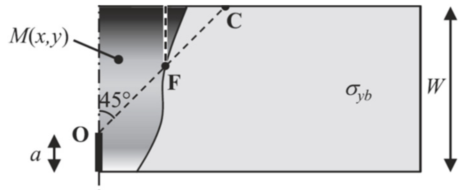

Figure 3: The slip line originating from the notch tip is shown along the OFC and the equation to calculate equivalent mismatch (M_{eq}) is given (Hertelé, De Waele et al. (2014), Hertelé, O'Dowd et al. (2015))

3. Numerical model

The representative weld shown in *figure 2* is modelled using Finite Element (FE) technique in ABAQUS software v6.11 *(figure 4)*. The SE(T) simulations were performed under 2D plane strain conditions with clamped end (no rotations allowed) on one side and a displacement of *2mm* applied on the other end. Specimen dimensions are *L=150mm*, *W=15mm,* and a blunt crack tip was modelled with initial radius *r=0.005mm*, closely approximating a perfectly sharp crack. The material behavior is elastic-perfectly plastic. Three-dimensional, eight node linear elements with reduced integration have been used. This is similar to the model used in Hertelé, De Waele et al. (2014), except for the perfectly plastic condition. In order to validate analytical lower and upper bound equations using limit loads obtained from simulations, four different specimen configurations are chosen. The chosen geometries are shown in *figure 5***.** The geometrical and material properties for root and cap of the weld material and base material were chosen such that a wide range of configurations is assessed. In the configurations (a)-(d) shown in *figure 5*, the parameters were chosen as *a/W= 0.2;0.4,* $\beta = 10;30$ and $M_r = 0.85;1.00;1.15$. The difference in weld root and cap mismatch is characterized by the equation $\Delta M_{cr} = M_c - M_r$ where $\Delta M_{cr} = 0.00;0.15;0.30$. The weld cap strength is always equal to or higher than the weld root strength, representing realistic welding practice. All the base material properties were kept constant at $E/\sigma_{yb} = 400$ and $\sigma_{yb} = 500MPa$. These variations of parameters help in validating the limit loads estimated by analytical equations for different cases of weld heterogeneity.

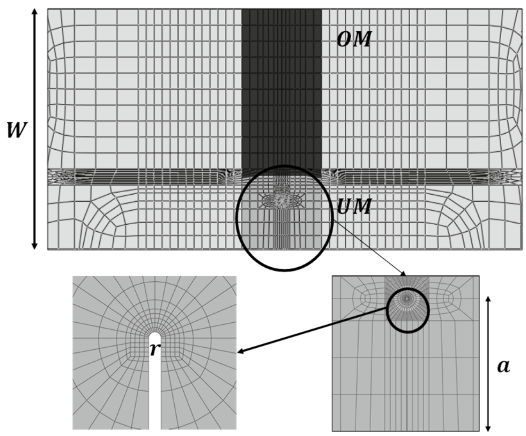

Figure 4: A part of SE(T) numerical model showing the regions of undermatch and overmatch, meshing technique and the notch

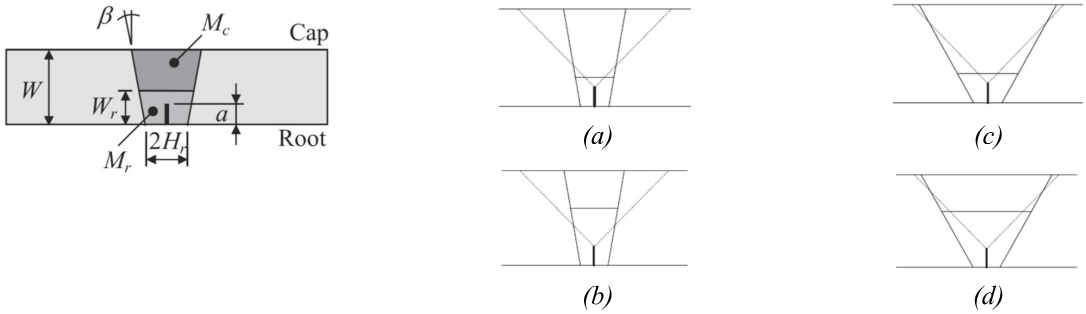

Figure 5: The varying parameters of the weld and the chosen configurations (a)-(d) (Hertelé, De Waele et al. (2014))

4. Results and discussions

The analytical lower bound equations *(sect. 2.1)* and upper bound solution (*sect. 2.2*) have been validated using SE(T) numerical model. The material properties for root and cap of the weld material and base material were chosen such that a wide range of characteristics are assessed. *Figure 6* shows the comparison of FE and analytical results for different weld configurations. Analytical results are plotted on ordinate and numerical results are plotted on abscissa.

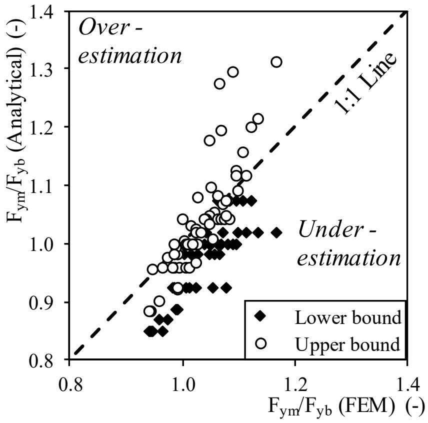

Figure 6: Analytical and numerical limit load plots of welded SE(T) specimen have been plotted. The regions of over-estimation (non-conservative/unsafe) and the under-estimations (conservative/safe) are also depicted

From figure 6, it can be seen that the lower bound estimates lie lower than the 1:1 line which indicates that the estimate provided by the equation (2) is lower than the actual limit load of the structure (as one would expect). The differences between the predicted F_{ym}/F_{yb} analytical and numerical solutions were up to 15%. The average difference of all points is 6% and lies in the region of under-estimation. In spite of the different material properties of the root and cap of the weld along with different notch configurations, the estimations remain lower bound and thus justifying the developed analytical lower bound equation. The difference between the 1:1 line and the predicted lower bound value is the highest when the M_c was equal to 0.85. This points out to the fact that the differences are higher when the entire weld is weaker than the base metal. Similarly, the upper bound estimates were plotted in *figure 6* and is compared with the 1:1 line. The trend is observed in the estimated values were similar to the results obtained by Hertelé, De Waele et al. (2014). However, several values were underestimated, even though upper bound equations were used. These results were observed in the specimen configurations (b) and (d) shown in *figure 6*. Predominantly, for specimen (b), limit loads were mostly underestimated due to the fact that the effect of the weld cap is not observed as the slip line does not penetrate through the cap region. Due to this, the analytical solution does not take into account any effect of the cap on limit load while the FE simulation, though minimal, incorporates its effect. In specimen (d), the values were underestimated when the mismatched weld region is undermatching and the upper bound equations are mainly developed for overmatching specimens. This was observed in other specimen configurations too. The others show a good correspondence with the 1:1 line with errors less than 5%. By observing the F_{ym}/F_{yb} values obtained by lower bound and upper bound equations, the upper bound values are closer to the actual values i.e. 1:1 line while the lower bound values differs by up to 15%. From this observation, it is evident that the lower bound equation provides a safer and conservative assessment of the limit loads which assists in the effective design of a welded structure. The outcome of this work motivates for the further assessment of limit loads of complex weld configurations for SE(T) specimens.

5. Conclusions

In this research, theoretical lower and upper limit load solutions of a heterogeneous welded connection under tension have been compared against numerical results in order to validate the accuracy of the theoretical formulations. Lower Bound (LB) solutions have been derived for heterogeneous welded SE(T) specimen considering three possible failure modes. An already available Upper Bound (UB) equation was implemented. Several weld configurations are modelled numerically, assuming elastic-perfectly plastic materials. Limit load was obtained by considering the maximum load attained by the SE(T) simulation for the applied displacement. The main outcomes of this research for the considered set of weld configurations are as follows:

- The lower bound approach provides lower estimates of limit loads than the FE results for a wide range of weld material and specimen configurations.
- The LB limit load solutions provides results lower than the actual limit loads up to 15% and thereby contributing to conservatism.
- The UB limit load solutions are generally close to the simulated limit loads, but differences increase as the weld strength mismatch increases. There are cases where the upper bound limit loads are contradictorily exceeded by the simulated values. These cases are subject to further examination.

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